JOURNAL OF AIRCRAFT
Vol. 42, No. 2, March–April 2005

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Two Simplified Versions of Supersonic Area Rule

Vojin R. Nikolic*
Minnesota State University, Mankato, Minnesota 56001
and
Eric J. Jumper†
University of Notre Dame, Notre Dame, Indiana 46556

Introduction

SINCE its formulation, the supersonic area rule has been used extensively in the design of high-speed airplanes. It has proven itself as a good approximation to the exact linearized theory result by Lomax. This method is well suited for investigating the effects of the shape and location of aircraft components and external stores on the wave drag of the configuration. The late 1960s and 1970s witnessed a surge in the use of the method as a result mostly of the extensive exploration of the supersonic transport. With the recently renewed interest in supersonic airliners and business jets, this method seems likely to regain significance in the future, despite the progress that has been made in computational fluid dynamics. The major advantage of the supersonic area rule lies in its flexibility in use for both analysis and design tasks. However, because of its complexity and extensive input format, simplified methods requiring easier-to-prepare inputs have been sought. This paper presents two such methods.

Supersonic Area Rule

Supersonic area rule is a numerical method for prediction of zero-lift wave drag. Briefly stated, it involves determining[1]

$$D_w = \frac{1}{2\pi}\int_0^{2\pi} D(\theta)\,\mathrm{d}\theta \tag{1}$$

with

$$D(\theta) = -\frac{\rho V^2}{4\pi}\int_{x_A(\theta)}^{x_B(\theta)}\int_{x_A(\theta)}^{x_B(\theta)} S''(x_1)S''(x_2)\,\ln|x_1 - x_2|\,\mathrm{d}x_1\,\mathrm{d}x_2 \tag{2}$$

where D_w is the zero-lift wave drag; θ the angle between the y axis and the projection of the normal to the Mach plane onto the y–z plane; θ is positive in the positive y–z quadrant; ρ, V the density and velocity, respectively, of the freestream; $S(x_i)$ the area of the projection of the intersection of the airplane configuration by the plane defined by x_i; and μ and θ onto the y–z plane, where $\mu = \sin^{-1}(1/M)$ is the Mach angle.

Equation (2) represents the well-known Jones result or the supersonic area rule. The striking resemblance of Eq. (2) with the well-known slender-body wave-drag equation given by von Kármán[2] is noted. However, the restrictions for the two equations are quite different.

The integral of Eq. (2) needs to be evaluated for many values of θ between 0 and 360 deg or because aircraft are typically symmetric, between 0 and 180 deg. If a 1-deg increment is adapted, then the integral needs to be evaluated 180 times. Finally, the wave drag at a Mach number is evaluated by the use of Eq. (1). A complete description of the implementation procedure is presented in Ref. 3. Several computer programs for implementation of the supersonic area rule have been written. The most widely used program is that developed by the Boeing Company and described by Harris.[4] This computer program was included as a part of a more complex design procedure by Baals et al.,[5] where they showed that for conventional configurations of supersonic aircraft the supersonic area rule gave good results.

The first author used Harris's program in the development of a supersonic fighter aircraft while at the Aeronautical Institute, Belgrade. The aircraft was designed to attain a Mach number of 1.8+ at an altitude and was designed with the help from two leading European aerospace manufacturers, British Aerospace and Marsell Dassault. While the British and French design teams used other, more complex methods, the wave-drag results agreed very well, typically within 3%.

Simplifications to the Supersonic Area Rule

Bearing in mind the calculation complexity and the difficulty of preparing appropriate input data for the standard supersonic area, it is not surprising that aerodynamicists have tried to simplify the method. Harris[4] made a modification by simplifying the fuselage description. Smith et al. described a simplification that used one set of the Mach planes—that of parallel vertical planes which intersect the configuration planform along Mach lines.[6]

Jumper's Simplification

The most fundamental simplification made to the supersonic area rule is that one proposed by Jumper.[7] Instead of describing the actual aircraft configuration by inputting a large number of points in a three-dimensional space, Jumper proposed entering data for the normal plane cut areas along the aircraft longitudinal axis. These make up the equivalent body of revolution to which Eq. (2) is applied once for each Mach number of interest—and only once because for a body of revolution Mach plane cut areas do not depend on a particular roll angle θ. This modification achieved two important simplifications: First, the input data set becomes the simplest possible, and secondly, only one set of Mach planes need be used for each Mach number of interest. Jumper did not propose his method for highly accurate wave-drag predictions, but rather as an auxiliary tool for system-design studies or early program management decisions. As such, it is meant to supply the user with quick yet reasonable data, particularly if applied to predict the wave-drag increment as a result of adding near-fuselage-axis protuberances where a good deal of information about the aircraft in question is known.

To validate the simplification proposed, Jumper applied his procedure to a number of controls.[7] He showed that the simplified

Received 18 December 2003; revision received 27 May 2004; accepted for publication 28 May 2004.

*Professor, Department of Mechanical Engineering. Senior Member AIAA.
†Professor, Department of Aerospace and Mechanical Engineering. Fellow AIAA.

supersonic area rule exactly predicted the most favorable longitudinal location to place a concentric protuberance on a given body of revolution. As he has pointed out, that result could not be attributed to his simplification but to the full supersonic area rule because the simplification, when applied on a body of revolution, is no longer simplification. Then he applied the procedure to three very distinct configurations, including a body of revolution, an aircraft configuration involving moderately large area located far off axis, and an aircraft model representing an extreme case of large areas located far off axis. Supersonic area rule did a good job for the first and second configurations, but it began to fail on the third one.

The results pertaining to the body of revolution were identical, as expected, to those from the full supersonic area rule. The results for second model were within a 20% accuracy at $M = 1.1$ and within 7% at $M = 1.5$, which can be considered a good result keeping in mind the simplification proposed. And, finally, the results for the third configuration were far off the experimental ones as were those from the standard supersonic area rule.[7] Based on these results, Jumper concluded that, for those configurations for which the standard supersonic area rule gives good agreement with experiment, it can be expected that his modified procedure would be able to predict the wave drag with reasonable accuracy.

Finally, Jumper applied the simplified method to predict the wave-drag increments as a result of the addition of two conformal pallets to the F-15 aircraft. These increments were then compared to both wind-tunnel data and supersonic area rule results. These results will be discussed next; for now it will suffice to say that they were better than expected. This meant that this fast procedure should be further exploited.

Four New Proposed Simplifications

Four new schemes are derived from the following consideration. The flowfield around a three-dimensional body moving at supersonic speeds is conical in nature rather than planar. Thus, the following approach seemed worth exploring: Instead of using planes at different roll angles tangent to the characteristic Mach cone, let us for the same purpose employ the cone itself.[8] At a fixed Mach number, two Mach cones can originate at a point: one having the generators directed downstream and the other upstream. By making use of these two cones, four simplified approaches could be conceived.

Let us start, as Jumper did, with a whole aircraft replaced by a single body of revolution having the same longitudinal cross-sectional area distribution as the aircraft (see Fig. 1). The length of the aircraft, or the body, L is divided into n equal segments, Δx each. Next a Mach number of interest, say M_1, is chosen. Referring to Fig. 1, this Mach number defines a Mach cone with a half-angle $\mu_1 = \sin^{-1}(1/M_1)$. The cone can be originated at any point on the

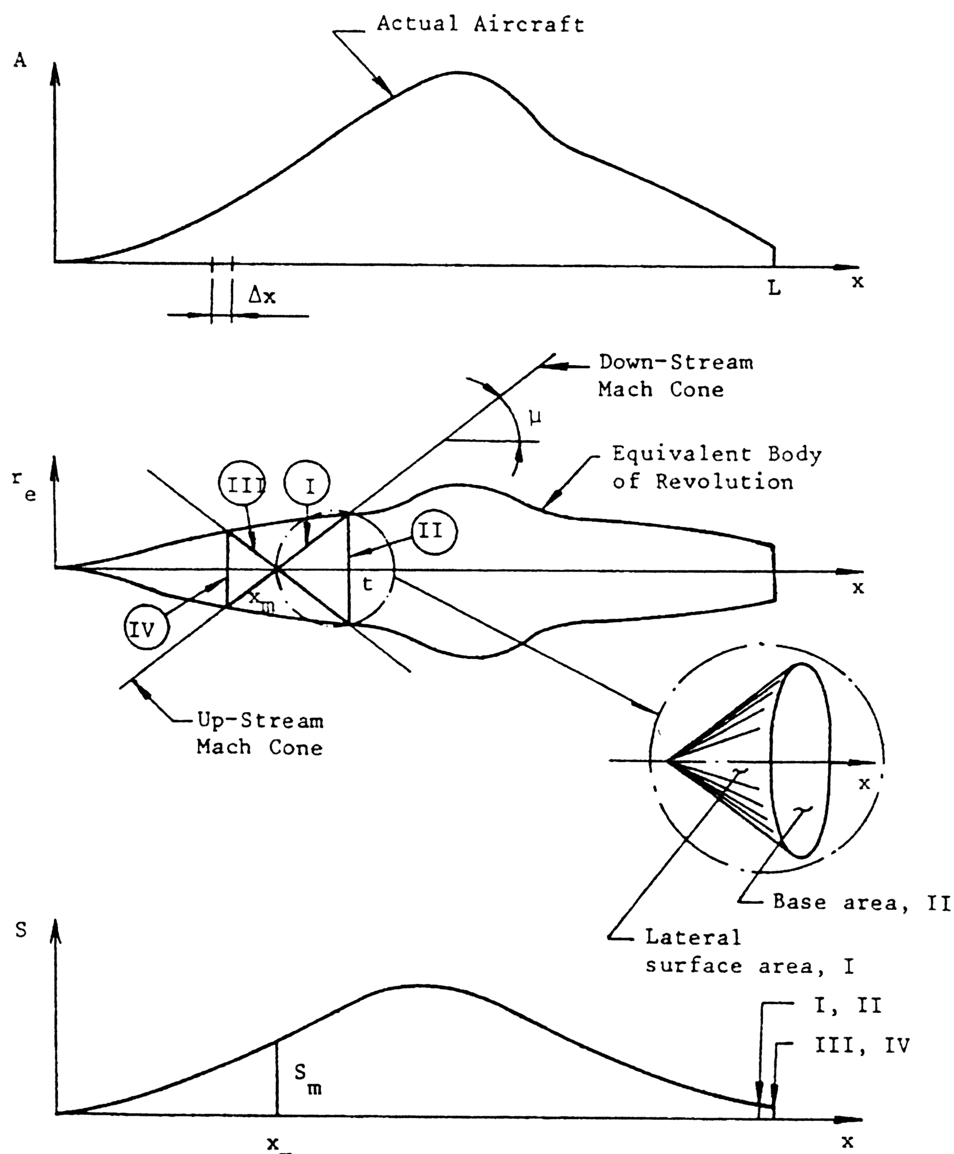

Fig. 1 Four new proposed simplifications.

body axis in either direction—downstream or upstream. Let us consider the downstream cone the vertex of which is at $x_m = m\Delta x$, where $m = 1, 2, \ldots, n-1$. The cone generators intersect the body of revolution making a cone with a height given by $(t - x_m)$. The lateral surface area of the cone, denoted in Fig. 1 as I, is designated S_m. Next the cone is moved downstream through Δx and its vertex reaches point $x_{m+1} = (m+1)\Delta x$, and a new area S_{m+1} is obtained in the same manner. Once the cone vertex has traveled through all of the points starting from x_1 and finishing at x_{n-1}, a set of areas $(S_1, S_2, \ldots, S_{n-1})$ has been obtained. This set is then used to construct another equivalent body of revolution such that the cross-sectional area formed by planes normal to the x axis of this body at any x_m is exactly S_m. Then Eq. (2) is applied to this body, and the wave-drag coefficient is calculated using a procedure similar to that described earlier. Another Mach number, say M_2, defines a new Mach cone, and the procedure is repeated yielding the wave-drag coefficient that corresponds to M_2.

Because the area designated by I in Fig. 1 was employed, this scheme is referred to as version I. Version I does not employ any kind of projections of the cone surface area in obtaining the S_m. Because the standard supersonic area rule employs the projections of S_m onto a plane normal to the body axis, a new set of projected areas can be defined and is marked by II on Fig. 1. When used in conjunction with Eq. (2), these areas yield a drag coefficient referred to as version II. It is now easy to define two more versions, III and IV.

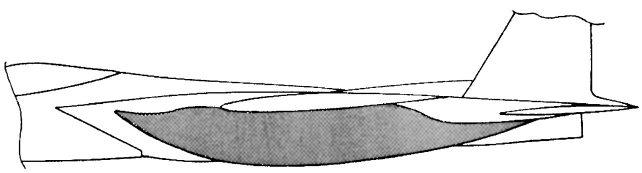

Fig. 2 Location of the T-94 pallet.

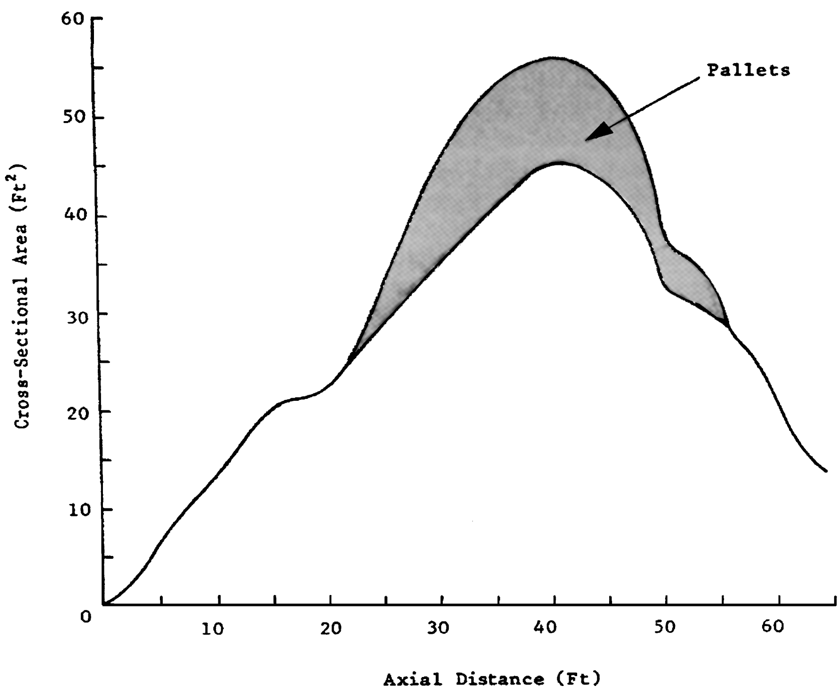

Fig. 3 Area distributions of F-15 with and without fast pack pallets.

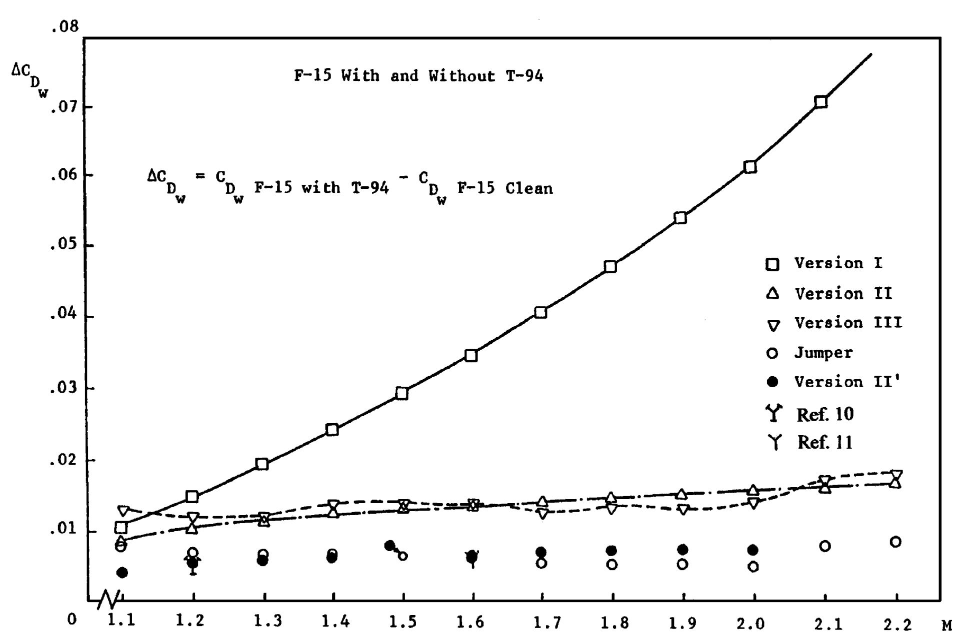

Fig. 4 Incremental drag caused by adding pallets.

Those are obtained basically the same way as versions I and II—the only difference being that the upstream Mach cone was used.

Because the four schemes just described shared the same starting point as Jumper's simplification (i.e., reducing a complex aircraft structure to a simple body of revolution), they preserved the two simplifying features of his modification—a simple input data format and a need to perform only one integration for a Mach number of interest. Thus, there was no need to employ complex procedures such as three-dimensional approximation and curve-fitting techniques as required in application of the full supersonic area rule algorithms (for example, see Ref. 9), where a 24-term Fourier series was used to calculate the slope of the area distribution.

Results and Discussion

The four versions just described were employed to find the wave drag of a number of aircraft configurations. Also the Jumper method was used for comparison. About 20 different configurations were found in the literature, which could serve as test cases. Several of these aircraft were chosen for use in the first phase.[3] After the first few aircraft were analyzed, it became clear (see the following) that version II was the only one among the four new proposed methods giving appropriately behaved C_{Dw} vs M curves. It was further noticed that this method consistently overpredicted wave drag by a factor of two. Therefore, the results were arbitrarily multiplied by one-half (version II'). The results of all four versions initially, then version II' only, and Jumper's method were compared to free flight data, wind-tunnel, and the standard supersonic area rule results whenever available. Because of limited space, only the results pertaining to three of these cases will be discussed here. The rest will be presented in a separate paper.

F-15 with and Without Conformal Pallets

The two configurations of the aircraft are described in Ref. 7. Figure 2 shows the conformal pallet T-94 placed under the left wing of an F-15 aircraft, and Fig. 3 shows the area distributions. These configurations were run employing the four new proposed simplifications. The results of these tests along with those reported by Jumper[7] are presented in Fig. 4, which shows the difference between the wave drag of the aircraft with the pallets in place minus the wave drag of the clean aircraft. The results obtained by employing version IV are not included; this version produced unrealistically high C_{Dw} values. The only new proposed version that gave the expected shape of the C_{Dw} vs M curve was version II. The results obtained by employing version II' looked reasonable. These results were close to those from Jumper's study, which, in turn, were in excellent agreement with wind-tunnel data available. Two values from Refs. 10 and 11 almost coincided with the version II' results at $M = 1.2$ and 1.6.

F-105 with and Without Rear-Body Bump

Description of the aircraft model can be found elsewhere.[12–14] Wind-tunnel results were available for $M = 0.60$ to 1.13 and $M = 2.01$. Several configurations have been tested; however, the only configurations for which the longitudinal cross-sectional area distribution data were available were the baseline one and the one with a rear-body bump added to improve the airplane transonic drag (see Fig. 5).[12]

A summary of the results is given in Table 1 containing the drag differences. The justification for such approach was that at zero angle of attack no flow separation would occur, and therefore the drag difference was basically caused by the wave-drag difference. It can be seen from Table 1 that Jumper's simplification was the only modification able to give reasonable results, achieving a 14% accuracy at $M = 1.13$ and a 9% accuracy at $M = 2.10$. This agreement is believed to be caused by the way in which volume of the bump was added to the basic configuration, namely, as a concentrically placed volume increment, which represents the most favorable case for Jumper's simplification. It is recalled that his simplification was primarily developed for investigation of near-to-axis-protuberances.

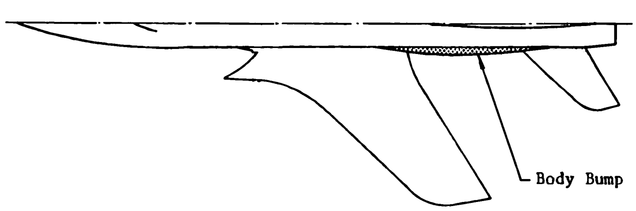

Fig. 5 F-105 with and without rear-body bump.

Northrop F-5E (Single-Seat) and F-5F (Two-Seat) Versions

The data for F-5E (Ref. 8) were the most complete for the purpose of this study because wave-drag coefficients over a range from $M = 1.0$ to 1.8 were available from two sources—the 124J Wave Drag Program developed by Northrop and the Langley Wave Drag Program. The Langley program is generally used by the aircraft companies in the United States. The relative errors, based on the Langley program results, are shown in Fig. 6. It can be seen from Fig. 6 that the relative errors for both simplified methods stayed

Table 1 F-105 with and without rear-body bump (zero-lift drag comparison) $\Delta C_{Dw} = C_{Dw,Bumpoff} - C_{Dw,Bumpon}$ (ΔC_{Dw} from)

M	I	II	III	IV	Jumper	II'	NACA
1.13	0.001	0.0008	−0.0017	−0.0013	0.00198	0.0004	0.0023
2.01	0.0136	0.0034	0.0236	0.0058	0.00109	0.0017	0.0010

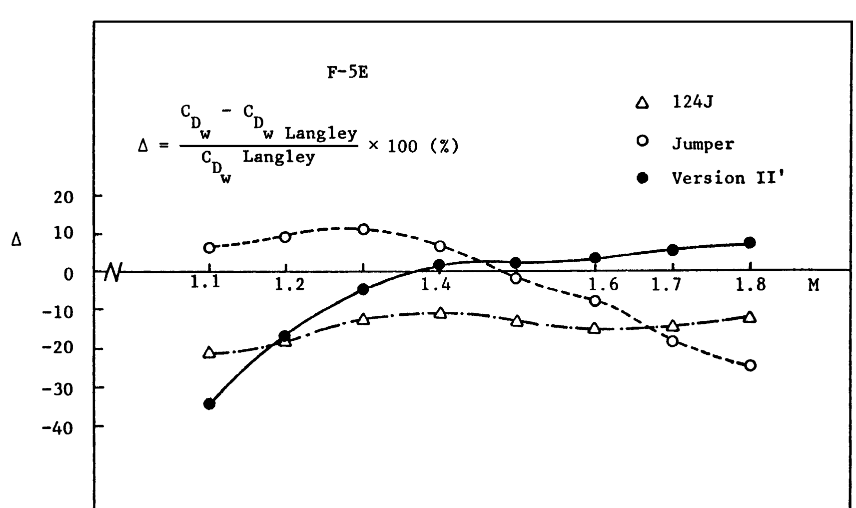

Fig. 6 Relative errors in wave-drag data for F-5E aircraft.

within limits of 10% over a wide range of Mach numbers—Jumper's method being superior at lower Mach numbers, up to $M = 1.25$, and version II' over the rest of the range considered.

The data for the F-5F were not as complete as those for the F-5E, and wave drag had to be estimated from the minimum-drag coefficient. Jumper's simplification gave good results at Mach numbers up to $M = 1.35$—within 10% accuracy. The results obtained from version II' correlated to both flight-test data and those estimated by Northrop better at higher Mach numbers, $M = 1.40$ to 1.80. The complete results of these tests will be presented in a separate paper.

Conclusions

The Jones formula, known as the supersonic area rule, has been a major tool in the design and analysis of supersonic aircraft since its introduction. The complexity of the required input format, as well as the calculations involved, has been pointed out. Simplifications to the procedure have been sought. The Jumper simplification has been the first major modification of the standard supersonic area rule. The study further validated the Jumper method and explored other possibilities for simplification. Four such methods were developed, one of which gave interesting results. The method uses cuts with the downstream Mach cone lateral surface on the equivalent body of revolution. Three cases of actual aircraft and wind-tunnel models were used for numerical validation of both the Jumper method and the new proposed simplification. Some promising results were found along with tremendous decreases in input preparation and computing time. The new proposed modification correlated very well at moderate supersonic Mach numbers. The Jumper method proved superior to the new simplification at transonic and lower supersonic speeds. Further investigations of these methods are necessary prior to their general acceptance for quick and reasonable accurate zero-lift wave-drag calculations.

References

[1]Jones, R. T., "Theory of Wing-Body Drag at Supersonic Speeds," NACA Report 1284, Jan. 1956.

[2]Liepman, H. W., and Roshko, A., *Element of Gas Dynamics*, Dover, New York, 2002, pp. 235–239.

[3]Nikolic, V. R., and Jumper, E. J., "Zero-Lift Wave Drag Calculations Using Supersonic Area Rule and Its Modifications," AIAA Paper 2004-217, Jan. 2004.

[4]Harris, R. V., Jr., "An Analysis and Correlation of Aircraft Wave Drag," NASA TM X-947, March 1964.

[5]Baals, D. D., Robine, A. W., and Harris, R. V., Jr., "Aerodynamic Design Integration of Supersonic Aircraft," *Journal of Aircraft*, Vol. 7, No. 5, 1970, pp. 385–394.

[6]Smith, N. F., Brilat, R. P., and Guy, L. D., "Drag of External Stores and Nacelles at Transonic and Supersonic Speeds," NACA RM L531I23b, Oct. 1953.

[7]Jumper, E. J., "Wave Drag Prediction Using a Simplified Supersonic Area Rule," *Journal of Aircraft*, Vol. 20, No. 10, 1983, pp. 893–895.

[8]Nikolic, V. R., "An Investigation of New Possibilities to Simplify the Standard Supersonic Area Rule," M.S. Thesis, Dept. of Aeronautics and Astronautics, Air Force Inst. of Technology, Dayton, OH, Dec. 1983.

[9]Whitcomb, R. T., "A Study of the Zero-Lift Drag-Rise Characteristics of Wing-Body Combinations near the Speed of Sound," NACA Rept. 1273, Jan. 1956.

[10]Lemley, C. E., Triplett, W. E., and Verhoff, A., "Aerodynamic Interference due to Optical Turrets," Air Force Wright Aeronautical Labs., Flight Dynamics Lab., Rept. AFWAL-TR-80-3058, Dayton, OH, Sept. 1978.

[11]Meyer, W. L., "Summary of Results of Series XI Polysonic Wind Tunnel Tests on a 4.7% Scale Model of the Model 199A-PSWT Test #366," McDonnell-Douglas Aircraft Co., Rept. MDC 4589, St. Louis, MO, Dec. 1976.

[12]Luoma, A. A., "Investigation of a 1/22-Scale Model of the REPUBLIC F-105 Airplane in the Langley 8-Foot Transonic Tunnel; Lateral, Directional, and Longitudinal Static Stability and Control," NASA RM SL57H06, Aug. 1957.

[13]Luoma, A. A., "Investigation of a 1/22-Scale Model of the REPUBLIC F-105 Airplane in the Langley 8-Foot Transonic Tunnel; Static Longitudinal Static Stability and Control and Performance Characteristics at Transonic Speeds," NASA RM SL56D12, April 1956.

[14]Spearman, M. L., Driver, C., and Robinson, R. B., "Aerodynamic Characteristics of Various Configurations of a Model of a 45 Degree Swept-Wing Airplane at a Mach Number of 2.01," NASA RM L54J08, Oct. 1955.